

# Is the super-Penrose process possible near black holes?

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We consider collisions of particles near generic axially symmetric extremal black holes. We examine possibility of indefinitely large extraction of energy (the so-called super-Penrose process) in the limit when the point of collision approaches the horizon. Three potential options are considered (fractional powers of the lapse function in the relations between the energies and the angular momenta of particles in the point of collision), collision between outgoing particles and ingoing ones, collision in the ergoregion far from the horizon). It turns out in all three cases that states suitable for the super-Penrose process cannot be obtained from the previous collision of particles with finite masses and angular momenta.

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## I. INTRODUCTION

Collisions of two particles near a black hole can lead to unbound energy  $E_{c.m.}$  in their centre of mass, provided one of them is fine-tuned. This observation made in Ref. [1] for extremal Kerr black holes, stimulated significantly interest to the energetics of particle collisions in strong gravitational field. At first, the main part of researches concentrated on the question for which objects and under which condition large  $E_{c.m.}$  are possible thus extending the results of [1] to more general types of spacetimes and scenarios. Meanwhile, there is another question which is under active discussion just now. Whether or not it is

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possible to gain unbound Killing energy  $E$  of debris measured at infinity after collision and thus obtain unbound energy extraction? In several works it was shown that for collisions near black holes, in spite of unbound  $E_{c.m.}$ , the energy  $E$  turns out to be quite modest. This was shown for the Kerr metric in [2], [3] and arbitrary "dirty" (surrounded by matter) black holes in [4]. These results were obtained for collisions between ingoing particles sent to a black hole from infinity.

Meanwhile, there exist more involved scenarios. It was noticed in [5] numerically that collision between fine-tuned outgoing particle and ingoing not fine-tuned ("usual") one in the Kerr background leads to rather significant amplification of the energy  $E$ . This result was confirmed analytically in [6], [7]. Moreover, it was noticed in [8] numerically and in [9] analytically that head-on collision between two usual particles near black holes gives rise to unbound  $E$  (it was called super-Penrose process in [8]). The problem, however, consists in that it is difficult to create an outgoing usual particle in the immediate vicinity of a black hole where all particles with finite energy should be ingoing. One can try to get a usual particle near the horizon as a result of previous collision but it turns out that for finite masses and angular momenta of initial particles this is impossible not only for the Kerr metric [10] but in much more general case [11].

However, previous studies of collisions near black holes do not exhaust all possible cases since there are three additional potential options for getting unbound  $E$  not explored yet. (i) One can try to weaken requirement on the type of particles and consider the case intermediate between usual and fine-tuned ("critical") particles. Namely, one can probe such a deviation from the special relation between the energy and angular momentum that includes fractional powers of the lapse function at the point of collision. (ii) The conclusion about serious (with the factor about 14) but still restricted amplification of the energy due to collision was obtained for the Kerr metric only. It is of interest to consider generic axially symmetric stationary black holes, not specifying their metric and to elucidate, whether or not the unbound  $E$  are possible. (iii) There are scenarios in which collision occurs not near the horizon but somewhere in the ergoregion [12], [13], provided the angular momentum of either particle is large negative. It was shown quite recently that the super-Penrose process for such scenario is possible by itself [14]. However, the question arises, how to prepare a state with initial large angular momentum. More precisely, can it be obtained in the preceding collision with all finite characteristics of particles?

In the present work, we discuss all three aforementioned issues. It turns out that in all three cases the answer is negative that gives new restrictions on physical realization of the super-Penrose process (although it is not excluded in principle). We assume that there is no interaction between particles and the electromagnetic field. Throughout the paper, we put fundamental constants  $G = c = 1$ .

## II. BASIC EQUATIONS

We consider the metric

$$ds^2 = -N^2 dt^2 + g_\phi(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_\theta d\theta^2, \quad (1)$$

in which the coefficients do not depend on  $t$  and  $\phi$ . Correspondingly, the energy  $E$  and angular momentum  $L$  are conserved. In the present work, we will be interested in the equatorial motion. Then, the geodesic equation of motion gives us

$$m \frac{dt}{d\tau} = \frac{X}{N^2}, \quad (2)$$

$$m \frac{d\phi}{d\tau} = \frac{L}{g_\phi} + \frac{\omega X}{N^2}, \quad (3)$$

where  $m$  is the mass,  $X = E - \omega L$  and  $\tau$  is the proper time. The forward-in-time condition reads  $\frac{dt}{d\tau} > 0$ , whence

$$X > 0 \quad (4)$$

everywhere for  $N \neq 0$ .

We assume that there exists the horizon on which  $N = 0$ . We assume that the horizon is extremal. If  $X_H = 0$  (that is compatible with the forward-in-time condition), we call a particle critical (hereafter, subscript "H" means that the corresponding quantity is taken on the horizon). If  $X_H \neq 0$  and is separated from zero, we call a particle usual. In what follows, we will need further classification. We call a particle near-critical if  $X_H = O(N_c)$  and, for brevity, fractional if

$$X_H = O(N_c^s) \quad (5)$$

with

$$0 < s < 1. \quad (6)$$

Hereafter, subscript "c" denotes quantities calculated at the point of collision.

We restrict ourselves by motion within the plane  $\theta = \frac{\pi}{2}$  where we scale the radial coordinate to achieve  $A = N^2$ . Using equation (3) and normalization condition for the four-velocity, one can obtain that the radial momentum

$$p^r = \sigma Z, \quad (7)$$

where

$$Z = \sqrt{X^2 - \alpha^2}, \quad (8)$$

$$\alpha^2 = N^2 \delta^2, \quad \delta^2 = m^2 + \frac{L^2}{g_\phi}, \quad (9)$$

$\sigma = -1$  for the ingoing particle and  $+1$  for the outgoing one.

Let two particles 1 and 2 collide to produce particles 3 and 4. Then, the conservation laws at the point of collision give us for the energy and angular momentum:

$$E_1 + E_2 = E_3 + E_4, \quad (10)$$

$$L_1 + L_2 = L_3 + L_4. \quad (11)$$

It follows from (10), (11) that

$$X_1 + X_2 = X_3 + X_4. \quad (12)$$

The conservation of the radial momentum reads

$$p_1^r + p_2^r = p_3^r + p_4^r. \quad (13)$$

In view of (7), (8), it is equivalent to

$$\sigma_1 Z_1 + \sigma_2 Z_2 = \sigma_3 Z_3 + \sigma_4 Z_4. \quad (14)$$

We assume that it is particle 3 that escapes to infinity.

### III. FRACTIONAL DEPENDENCE ON $N$

We deal with a black (not white) hole, so a typical particle that moves along the complete geodesics with finite nonzero energy cannot move away from a black hole in the immediate vicinity of the horizon (see below for details). However, if a particle is originated from some previous collision, this cannot be excluded in advance. Let such a particle ( $\sigma_1 = +1$ )

collide with the second one having  $\sigma_2 = -1$  (head-on collision). Let us consider the simplest situation when both particles have the same energy  $E_0$ , mass  $m$  and angular momentum  $L_0$ . It is also supposed that products of collisions have the same mass  $\mu$ . Then, it was found in [15] that the maximum possible energy of debris

$$E_{\max} = E_0 + \frac{(\omega X_0 + \frac{L_0 N^2}{g_\phi})\sqrt{g_\phi}}{N}\sqrt{B}, \quad B = 1 - \frac{\mu^2 N^2}{X_0^2 - \frac{N^2 L_0^2}{g_\phi}}, \quad (15)$$

where all quantities are taken at the point of collision. Eq. (15) was obtained in the context not connected with black holes. However, it is exact and is valid independently of the presence or absence of the horizon.

Let collision occur very nearly to the horizon, so  $N \rightarrow 0$ . If both particles are usual,  $X_H \neq 0$ , so  $E_{\max}$  grows like  $N^{-1}$  and the energy extraction from a black hole is unbound. This is in agreement with the previous numeric [8] and analytical [9] observations. However, the problem is that there exist severe restrictions on such a scenario and, for finite masses and angular momenta, it is impossible to create a usual particles in previous collisions [10], [11].

To find way out, one can try to weaken requirement on  $X$  and allow fractional particles. In general, near the extremal horizon, the following expansion is valid (see [16] for details) near the horizon:

$$X = X_H + B_1 L N + O(N^2), \quad (16)$$

where  $B_1 > 0$  is a constant. For fractional particles, it is the first term in (16) which dominates at the collision point due to (6). Then, it follows from (15) that  $E_{\max} \sim N_c^{s-1}$ . It still diverges, although more slowly than for usual particles.

Meanwhile, as is said above, such a scenario includes the outgoing particle in the immediate vicinity of the horizon that, in turn, requires preceding collision in which this particle was created. The most physically interesting state corresponds to two particles falling from infinity. At present, it is already known that in this case, the collision between a usual and near-critical particles near the horizon gives rise to a near-critical particle [2] - [4], so for both particles 1 and 3 the quantity  $X = O(N_c)$ . However, the case of fractional particles was not considered before. Below, we fill this gap and elucidate, whether or not it is possible to create the outgoing fractional particle in collision of two ones moving from infinity.

We consider such a scenario in which (i) particles 1 and 2 come from infinity, so  $\sigma_1 =$

$\sigma_2 = -1$ , (ii) particle 3 escapes to infinity. We imply that masses and angular momenta of initial particles are finite, so  $X_1$  and  $X_2$  are finite as well. Since  $X > 0$ ,  $X_3$  and  $X_4$  are also finite. We can choose at our will the finite quantities  $(X_1)_c$  and  $(X_2)_c$ . It is obvious from (14) that it is impossible to have  $\sigma_3 = \sigma_4 = +1$ , so at least one of particles 3 or 4 should have negative  $\sigma$ . We assume that  $\sigma_4 = -1$  but retain  $\sigma_3 = \pm 1$ . This allows to take into account the scenario in which particle 3 moves immediately after collision towards the horizon and bounces back.

Thus we have

$$-\sqrt{X_1^2 - \alpha_1^2} - \sqrt{X_2^2 - \alpha_2^2} = \sigma_3 \sqrt{X_3^2 - \alpha_3^2} - \sqrt{X_4^2 - \alpha_4^2}. \quad (17)$$

Now, different scenarios with fractional particles are considered separately. In doing so, we somewhat modify the approach developed in [3] for the Kerr metric and in [4] for dirty black holes.

#### A. Particle 1 is fractional, particle 2 is usual

Now,

$$(X_1)_c = \beta_1 N_c^q, \quad (18)$$

where  $0 < q < 1$ , the constant  $\beta_1 > 0$ .

Then, at the point of collision  $\alpha_1^2 = O(N_c^2)$ ,

$$Z_1 \approx X_1 - \frac{\alpha_1^2}{2X_1} \approx X_1 - CN^{2-q}. \quad (19)$$

Here  $C = \frac{1}{2\beta_1}(\frac{L^2}{g_\phi} + m_1^2)$ . It follows from (4) and (17) that particle 4 is usual and

$$X_3 + \sigma_3 \sqrt{X_3^2 - \alpha_3^2} \approx CN_c^{2-q}. \quad (20)$$

If  $\sigma_3 = +1$ ,

$$X_3 = O(N_c^{2-q}) = O(N_c^s), \quad (21)$$

$$\alpha_3 = O(N_c^{2-q}) = O(N_c^s), \quad (22)$$

where

$$s = 2 - q > 1 \quad (23)$$

in contradiction with (6). Thus this scenario is unsuitable for our goal.

If  $\sigma_3 = -1$ , there are two options. If  $\alpha_3 \lesssim X_3$ , estimates (21), (22) are still valid and the scenario should be rejected as well. If  $\alpha_3 \ll X_3$ , there is no turning point for particle 3, it falls into a black hole and cannot escape. Again, the scenario is unsuitable for our purposes.

### B. Particle 1 is fractional, particle 2 is near-critical

In the case under discussion,

$$(X_1)_c = \beta_1 N_c^{s_1}, \quad s_1 < 1, \quad (24)$$

$$(X_2)_c = \beta_2 N_c, \quad (25)$$

where  $\beta_1$  and  $\beta_2$  are positive constants. Now,  $(X_2)_c \ll (X_1)_c$  and gives negligible contribution into  $X$ . Then, the conservation law entails that at least one of particles has  $X$  of the order  $N_c^{s_1}$ . Let

$$(X_3)_c \approx \beta_3 N_c^{s_1}. \quad (26)$$

Then, eq. (12) entails

$$(X_4)_c \approx (\beta_1 - \beta_3) N_c^{s_1}, \quad \beta_1 > \beta_3. \quad (27)$$

As  $Z_{3,4}^2 > 0$ , we have

$$\alpha_{3,4} = \tilde{\alpha}_{3,4} N_c^{s_1}, \quad (28)$$

where  $\tilde{\alpha}_{3,4}$  are finite and, in general, nonzero.

Taking into account that  $s_1 < 1$ ,  $2 - s_1 > 1$ , we can omit the second term in the expansion (19). We also neglect  $Z_2$  in the conservation law (17) since it has the order  $N_c$ , whereas  $Z_1$  has the order  $N_c^{s_1}$ . Therefore, we have

$$-\beta_1 N_c^{s_1} \approx \sigma_3 \sqrt{X_3^2 - \alpha_3^2} - \sqrt{X_4^2 - \alpha_4^2}. \quad (29)$$

By substitution of (26) - (28), we obtain

$$\beta_1 = -\sigma_3 \sqrt{\beta_3^2 - \tilde{\alpha}_3^2} + \sqrt{(\beta_1 - \beta_3)^2 - \tilde{\alpha}_4^2} \equiv D. \quad (30)$$

Let  $\sigma_3 = +1$ . If  $\tilde{\alpha}_3 = \tilde{\alpha}_4 = 0$ ,  $D = \beta_1 - 2\beta_3 < \beta_1$ , so eq. (30) is contradictory. If  $\tilde{\alpha}_3 \neq 0$  or  $\tilde{\alpha}_4 \neq 0$ ,

$$D < \sqrt{\beta_3^2 - \tilde{\alpha}_3^2} + \sqrt{(\beta_1 - \beta_3)^2 - \tilde{\alpha}_4^2} < \beta_3 + \beta_1 - \beta_3 = \beta_1, \quad (31)$$

again in contradiction with (30).

For  $\sigma_3 = -1$ , eq. (30) can hold, provided  $\tilde{\alpha}_3 = \tilde{\alpha}_4 = 0$ . But if  $\tilde{\alpha}_3 = 0$ , there is no turning point for particle 3 and it falls down to a black hole and cannot escape.

Thus the present scenario is unsuitable for our purposes.

### C. Particle 1 is fractional, particle 2 is fractional

Now we have

$$(X_1)_c = \beta_1 N_c^{s_1}, \quad (32)$$

$$(X_2)_c = \beta_2 N_c^{s_2}. \quad (33)$$

In the expansion (19) (with  $q$  replaced with  $s$ ), the first and second terms coming from  $Z_1$  have the order  $N_c^{s_1}$  and  $N_c^{2-s_1}$ , respectively. The similar terms in  $Z_2$  have the order  $N_c^{s_2}$  and  $N_c^{2-s_2}$ . Let, for definiteness,  $s_1 < s_2$ . Then, the conservation law (17) entails in the main approximation just eq. (29). It follows from the conservation law (12) that

$$X_3 + X_4 = O(N_c^{s_1}). \quad (34)$$

As  $X_{1,2} > 0$  due to the forward-in-time condition (4), each of them has, in general, the same order. Then,

$$\alpha_{3,4} \leq X_{3,4} \approx \beta_{3,4} N_c^{s_1}, \quad (35)$$

$\beta_3$  and  $\beta_4$  are positive constants. Then, the main formulas from the previous scenario apply here with the same conclusion.

If  $s_1 = s_2$ , one should take into account in (17) contributions from both particles 1 and 2. Therefore, instead of  $\beta_1$ , the combination  $\beta_1 + \beta_2$  appears that can be denoted as a new constant  $\beta$ . Further, the previous formulas again apply. Thus this does not affect the conclusion.

## IV. KINEMATIC EXPLANATION

Here, we discuss qualitatively, what are the underlying kinematic reason that create obstacles to trajectories with usual [11] and fractional outgoing particles.



### A. Individual particle

For any particle moving in the axially symmetric stationary background, there exists the relation between the energy and angular momentum [17] at the given point with  $N = N_c$  :

$$X = N_c E_{loc}, \quad (36)$$

where  $E_{loc}$  is the energy measured by the local observer with the zero angular momentum (ZAMO [18]),

$$E_{loc} = \frac{m}{\sqrt{1 - V^2}}, \quad (37)$$

$V$  is the local velocity in the ZAMO frame.

Another physical characteristics, important in this context, is the proper time for motion between the point in the vicinity of the horizon where  $N = N_c$  and the point with a given  $N$  outside the horizon. It follows from (7) that

$$\tau = m \int_{r_c}^r \frac{dr}{Z}, \quad (38)$$

where  $N(r_c) = N_c$ . The horizon lies at  $r = r_+$  where  $N(r_+) = 0$ . In the present work we consider extremal horizons for which  $N \sim r - r_+$ . For nonextremal ones,  $N^2 \sim r - r_+$ .

For any finite  $E_{loc}$ , one obtains that  $\lim_{N_c \rightarrow 0} X = X_H = 0$ , so the particle becomes critical. By contrary, if  $X_H \neq 0$  (a usual particle), it entails

$$\lim_{N_c \rightarrow 0} E_{loc} = \infty, \quad (39)$$

so this energy becomes infinite, a particle approaches the horizon with the speed of light.

If such a particle moves in the inward direction, it reaches the horizon in a finite proper time both for nonextremal and extremal horizons since for usual particles  $Z \neq 0$  on the horizon. This is quite typical situation that occurs, in particular, for the Schwarzschild metric.

Let now a usual particle be outgoing. Then, it means that it crossed the horizon from inside a finite proper time ago and keeps moving further away from the horizon. But this is the situation of a white, not a black hole and should be rejected in the present context.

This difficulty does not arise if a particle has  $V < 1$  separated from zero. As the proper distance to the extremal horizon is infinite, the proper time between the horizon and any other point outside a black hole is now infinite (it diverges as  $-\ln N_c$  with  $N_c \rightarrow 0$ ). This

happens if  $X_H = 0$ , so the particle is critical. Such a particle could not arrive from the inner region and there is no contradiction with the nature of a black hole.

If a particle is fractional, it is seen from (8), (5), (38) that the proper time for traveling between a given point with  $r = r_c$  and the horizon  $r = r_+$  is finite since although  $X_H$  is small, it does not vanish. The same reasoning as in the case of usual particles applies here. Therefore, such individual particles cannot move in the vicinity of black (not white) holes in the outward direction.

### B. Restriction on particles created from collision and behavior of $E_{c.m.}$

If a particle appears as a result of collision, its trajectory starts just at the point of collision and cannot be extended into past. Therefore, the arguments based on the continuity of geodesics are not valid anymore. Kinematic reasonings cannot, in general, replace the detailed analysis of bookkeeping of energy and momenta. Meanwhile, it is also instructive to look at the problem in another way, considering the behavior of the energy in the centre of mass frame  $E_{c.m.}$ .

Let two particles 1 and 2 fall from infinity, collide and produce particles 3 and 4. Then,

$$E_{c.m.}^2 = -(m_1 u_{1\mu} + m_2 u_{2\mu})(m_1 u_1^\mu + m_2 u_2^\mu) = -(m_3 u_{3\mu} + m_4 u_{4\mu})(m_3 u_3^\mu + m_4 u_4^\mu), \quad (40)$$

$$E_{c.m.}^2 = m_1^2 + m_2^2 + 2m_1 m_2 \gamma_{12}, \quad (41)$$

where  $\gamma = -u_{1\mu} u_2^\mu$ . One can deduce from equations of motion that [19]

$$m_1 m_2 \gamma_{12} = \frac{X_1 X_2 - Z_1 Z_2}{N^2} - \frac{L_1 L_2}{g_\phi} \quad (42)$$

In a similar way, the same energy can be expressed in terms of characteristics of particles 3 and 4. Assuming that after collisions they move in opposite directions, one obtains that

$$E_{c.m.}^2 = m_3^2 + m_4^2 + 2m_3 m_4 \gamma_{34}, \quad (43)$$

$$m_3 m_4 \gamma_{34} = \frac{X_3 X_4 + Z_3 Z_4}{N^2} - \frac{L_3 L_4}{g_\phi}. \quad (44)$$

Using the conservation laws (12), (13) one can check that both expressions for  $E_{c.m.}$  coincide. Below we restrict ourselves by considering two cases. (The others can be considered in a similar way.)

### C. All particles are usual

If all particles are usual, eq. (41) gives us that for given masses,  $\lim_{N \rightarrow 0} E_{c.m.}^2$  is finite, whereas eq. (43) entails that it diverges like  $N^{-2}$ . This contradiction shows that the process under discussion is impossible for finite parameters of reaction.

However, the reaction becomes possible if one adjusts the mass of initial particle (say,  $m_1$ ) adjusted to  $N_c$  and allows it to diverge. Let  $E_1 = m_1$ ,  $m_3$  be finite. The conservation of energy gives us that  $X_4 \approx E_4 \approx m_1$ . Then, for large  $m_1$  one obtains from (41)

$$E_{c.m.}^2 \approx m_1^2. \quad (45)$$

It follows from (43) that

$$E_{c.m.}^2 \approx 4 \frac{m_1 X_3}{N_c^2}. \quad (46)$$

Comparing both expressions, one finds

$$m_1 \approx \frac{4X_3}{N_c^2} \quad (47)$$

in agreement with [10] and [11].

For the same reasons, two usual particles with finite masses falling from infinity cannot produce a near-critical outgoing particle.

### D. Collision of usual and fractional particles

Let us try collision between a usual and fractional particles falling from infinity. Equations (41), (42) and (19) entail that

$$E_{c.m.}^2 = O(N_c^{-s}). \quad (48)$$

Now, we expect that particle 4 is usual and particle 3 is fractional,  $X_3 = O(N_c^q)$ ,  $\alpha = O(N_c^q)$ , the value of  $q$  is found below. Eq. (43) gives rise to

$$E_{c.m.}^2 = O(N_c^{q-2}). \quad (49)$$

Comparing (48) and (49) we see that  $q = 2 - s$ . But in view of (6), we obtain that  $q > 1$ , so the attempt to find a scenario is unsuccessful. Thus evaluations of the energy in the centre of mass frame agrees with the analysis of the conservation law for the radial momentum.

## V. HEAD-ON COLLISION WITH PARTICIPATION OF NEAR-CRITICAL PARTICLE

It follows from the results above that it is impossible to create a fractional particle in preceding collision between two particles falling from infinity. Thus we return to the known situation when it is the near-critical particle that escapes to infinity after the first collision. But one may ask, whether or not its further collision with particles of other types can lead to the unbound energy. Now, let outgoing near-critical particle 1 collide with ingoing particle 2 and produces particle 3 that escapes to infinity. We assume that  $\sigma_4 = -1$ , so particle 4 falls into a black hole.

The conservation law (13) reads

$$\sqrt{X_1^2 - \alpha_1^2} - \sqrt{X_2^2 - \alpha_2^2} = \sqrt{X_3^2 - \alpha_3^2} - \sqrt{X_4^2 - \alpha_4^2} \quad (50)$$

We want to elucidate, whether or not we can obtain unbound energy  $E_3$ . If  $E_3 \rightarrow \infty$  but  $X_3$  is finite, it follows from (4) that

$$L_3 \rightarrow \infty. \quad (51)$$

To answer our question, we consider three cases separately.

### A. Particle 2 is usual

For small  $N_c$ ,

$$(X_1)_c \approx A_1 N_c, \quad (52)$$

$$(X_2)_c = O(1), \quad (53)$$

$A_1 > 0$  is a constant. It follows from (12) that one of produced particle is usual and another one is near-critical. Neglecting terms of the order  $N_c^2$ , we obtain from (50) that it is particle 3 which is near-critical, while particle 4 is usual and at the point of collision,

$$A_1 N - X_2 \approx -X_4 + \sqrt{X_3^2 - \alpha_3^2}. \quad (54)$$

Taking into account (16) we obtain

$$(X_4)_c \approx (X_2)_c + O(N_c), \quad (55)$$

and

$$\sqrt{(X_3^2)_c - \alpha_3^2} = O(N_c). \quad (56)$$

In turn, this entails that

$$\alpha_3 \leq (X_3)_c = O(N_c). \quad (57)$$

Then, it follows from (9) that  $L_3$  is finite. This is in contradiction with (51), so we cannot obtain unbound  $E_3$ .

### B. Particle 2 is near-critical

Then, in addition to condition (52), we have the similar one  $(X_2)_c \approx A_2 N_c$  instead of (53). The conservation law (50) gives us that also  $(X_3)_c \approx A_3 N_c$  and  $(X_4)_c \approx A_4 N_c$  with constants  $A_3$  and  $A_4$ . Then, using (57) we obtain that  $L_3$  is finite in contradiction with (51), so this scenario is incompatible with the super-Penrose process.

### C. Particle 2 is fractional

Now, particle 1 is near-critical, particle 2 is fractional. This is completely similar to the situation considered above. Now,

$$(X_1)_c = \beta_1 N_c, \quad (X_2)_c = \beta_2 N_c^q, \quad 0 < q < 1, \quad (58)$$

$$(X_3)_c \approx \beta_3 N_c^q, \quad (X_4)_c \approx (\beta_2 - \beta_3) N_c^q. \quad (59)$$

Then, eq. (50) gives us

$$-\beta_2 N_c^q = \sigma_3 \sqrt{(X_3)_c^2 - \alpha_3^2} - \sqrt{(X_4)_c^2 - \alpha_4^2}. \quad (60)$$

This equation coincides with (29) if  $\beta_1$  is replaced with  $\beta_2$ . The same analysis as before applies with the conclusion that the scenario is forbidden.

### D. When particles 3 and 4 move in the same direction

In this Subsection, we discuss the case  $\sigma_4 = \sigma_3$ . This is compatible with the general type of scenario under discussion. Let  $\sigma_3 = +1$ , so both particles 3 and 4 escape.

Then, we have from (14) that

$$\sqrt{X_1^2 - \alpha_1^2} - \sqrt{X_2^2 - \alpha_2^2} = \sqrt{X_3^2 - \alpha_3^2} + \sqrt{X_4^2 - \alpha_4^2}. \quad (61)$$

Here, particle 1 is near-critical by assumption. If particle 2 is usual we obtain obvious contradiction in the limit  $N_c \rightarrow 0$ . If particle 2 is fractional, its contribution still dominates for small  $N_c$ , and the contradiction arises again. Thus particle 2 should be near-critical. Then, it follows from (61) that particles 3 and 4 are also near-critical.

Let us write  $X_i = x_i N$ ,  $\alpha_i = \delta_i N$  ( $1 \leq i \leq 4$ ). Then, we have

$$\sqrt{x_1^2 - \delta_1^2} - \sqrt{x_2^2 - \delta_2^2} = \sqrt{x_3^2 - \delta_3^2} + \sqrt{x_4^2 - \delta_4^2}, \quad (62)$$

where all coefficients have the order  $O(1)$ .

According to (9), this entails that  $L_3$  and  $L_4$  are finite. This is inconsistent with (51), so that  $E_3$  is finite and the super-Penrose process is impossible.

The case  $\sigma_3 = \sigma_4 = -1$  can be considered in a similar way with the same conclusion.

## VI. COLLISIONS INSIDE ERGOREGION

In Ref. [12] another scenario was suggested for the Kerr metric (generalized in [13] for a generic metric (1)). Two particles collide in the ergoregion but not in the immediate vicinity of the horizon. One of them has the large negative angular momentum. Then, the energy in the centre of mass grows indefinitely. It was shown recently [14] that in this process  $E$  can be made as large as one likes, so the super-Penrose process is possible. However, the question remains how to create a particle with indefinitely large  $|L|$ . The most attractive option is to get it from some previous collision as was suggested in [12]. Below we examine this possibility and show that this is impossible.

Let two particles 1 and 2 with finite  $E_{1,2}$  and  $L_{1,2}$  collide. We assume that particle 4 has  $L_4 < 0$  with  $|L_4| \rightarrow \infty$ . Due to the conservation of energy and momentum,  $L_3 \rightarrow +\infty$ . Meanwhile,  $X_3$  and  $X_4$  remain finite since they are both positive and obey (12) where  $X_1$  and  $X_2$  are positive. In turn, this means that the condition  $Z^2 > 0$  with  $Z$  taken from (8) cannot be satisfied for particles 3 and 4 since  $X$  is finite, while  $L^2 \rightarrow \infty$ .

## VII. REMARKS ABOUT NONEQUATORIAL MOTION

Throughout the paper we considered equatorial motion only. More general case of nonequatorial motion is beyond the scope of the present paper, so we restrict ourselves by brief remarks. We expect no qualitative changes in this case for the following reasons. (i) In that case, in (9) one should simply replace  $m^2$  with  $m^2 + m^2 g_\theta \left(\frac{d\theta}{d\tau}\right)^2$ . In particular, near the horizon, where  $N \rightarrow 0$ , this changes the coefficients in the expansion (19) but does not affect general dependence of  $Z$  on  $N_c$ , so previous analysis of the conservation of the radial momentum applies. (iii) In general, the analysis gets more complicated since one should take into account the  $\theta$ -component of the momenta. However, this additional conditions can only restrict further the possibility of the super-Penrose process, so our negative results retain their validity.

## VIII. CONCLUSION

In the present work, we filled some gaps left from previous consideration. Its results in combination with those of the previous one [11] give a kind of a no-go theorem that applies to the collisional Penrose process. Namely, either (i) the state of initial particles does not lead to the super-Penrose process at all or (ii) it formally leads to it but such a state cannot be realized near a black hole from initial conditions with finite masses and angular momenta.

However, it does not forbid large (but restricted) extraction whose value is model-dependent. Say, for the Kerr metric the corresponding factor is about 14 [5] - [7].

One can also think of creating states, suitable for the super-Penrose process, not due to mechanical collisions but, say, from statistical fluctuations in thermal gas surrounding a black hole.

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